Exam Logical Verification 2014-2015

Thursday July 9, 2015, 12.00-14.45

6 exercises

Answers may be given in English or in Dutch.

Opgave 1. (*4+4+4+4 points*)
This exercise is concerned with first-order propositional logic (prop1) and simply typed λ-calculus ($\lambda\!\!\to$).

a) Show that the following formula is a tautology of minimal prop1:
$((A \to B) \to C) \to B \to C$.

b) Give the type derivation in $\lambda\!\!\to$ corresponding to the proof of 1a.

c) Give three different closed inhabitants in $\lambda\!\!\to$ of the following type:

$$B \to (B \to A) \to (B \to B) \to A$$

d) Explain the Curry-Howard-de Bruijn correspondence between terms in $\lambda\!\!\to$ and proofs in prop1 in detail.

Exercise 2. (*4+4+4+4+4 points*)
This exercise is concerned with first-order predicate logic (pred1) and λ-calculus with dependent types (λP).

a) Show that the following formula is a tautology of minimal pred1:
$\forall x. (P(x) \to (\forall y. P(y) \to A) \to A)$.

b) Give a λP-term corresponding to the formula in 2a.

c) Give a closed inhabitant in λP of the answer to 2b.

d) What is the type checking problem? Is it decidable for λP?

e) What is the type inhabitation problem? Is it decidable for λP?

Exercise 3. (*4+4+4 points*)
This exercise is concerned with second-order propositional logic (prop2) and polymorphic λ-calculus ($\lambda 2$).

a) Show that the following formula is a tautology of minimal prop2:

$$\forall a. ((\forall c. ((a \to c) \to c)) \to a)$$

b) Give the $\lambda 2$-term corresponding to the formula in 3a.

c) Give a closed inhabitant in $\lambda 2$ of the answer to 3b.

Exercise 4. *(4+4+5 points)*
This exercise is concerned with various typing and coding issues.

a) Give the polymorphic identity in $\lambda 2$.

b) First-order propositional logic can be defined in Coq as follows:

```
Parameter prop : Set.
Parameter imp : prop -> prop -> prop.
(* T expresses if a proposion in prop is valid
   if (T p) is inhabited then p is valid
   if (T p) is not inhabited then p is not valid *)
Parameter T : prop -> Prop.
```

Give variables modelling the introduction rule for implication and the elimination rule for implication.

c) What is the (impredicative) definition of the type of the natural numbers, given as Church numerals, in $\lambda 2$?

What is a term in this type that corresponds to the number 2?

Exercise 5. *(4+4+4 points)*
This exercise is concerned with inductive data-types in Coq. The constructors of the type `nat` are called `O` and `S`.

a) Give the definition of an inductive data-type `natpair` of pairs of natural numbers (with `nat` the type of natural numbers).

b) Give the induction principle for `natpair`.

c) Give two different definitions of data-types with zero elements.

Exercise 6. *(4+4+4+5 points)*
This exercise is concerned with inductive predicates in Coq. The constructors of the type `nat` are called `O` and `S`.

a) Consider the inductive predicate for less-than-equal in Coq:

```
Inductive le (n:nat) : nat -> Prop :=
| le_n : le n n
| le_S : forall m:nat , le n m -> le n (S m) .
```

Give if possible an inhabitant of the following, if it is not possible explain shortly why not:

```
le O (S O)
le (S O) O
```

b) Give the definition in Coq of an inductive predicate `tre` on natural numbers that holds exactly if the number can be divided by 3.

c) Give the Coq definition of an inductive predicate of type

```
even : nat -> Prop
```

that says whether a natural number is even or not.

d) Complete the following definition of conjunction in Coq:

```
Inductive and (A : Prop) (B : Prop) : Prop :=
```

The note for the exam is (the total amount of points plus 10) divided by 10.